# CHANGING THE CULTURE: DEVELOPING CREATIVE PROBLEM SOLVERS

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***Overheard in math class: "Oh God why is this happening to me? I go to church!"***

*"We have known for some years now…that most children's mathematical journeys are in vain because they never arrive anywhere, and what is perhaps worse is that they do not even enjoy the journey."*

Whitcombe, A. (1988), Mathematics creativity, imagination, beauty. *Mathematics in School, 17*, (2), 13-15

## Learning Mathematics

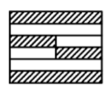
- "It doesn't make much sense. But, we are in math class, so I guess it does here," (sixth grader)[1]
- "In math, I do things just the opposite way from what I think it should be and it almost always works" (high school calculus student) [1]
- Constant emphasis on sequential rules and algorithms may prevent the development of creativity, problem solving skills and spatial ability[2]

[1] Mary Linquist, Past-President (1992-94) , NCTM, in her preface to *Making Sense: Teaching and Learning Mathematics with Understanding* (Hiebert et al., 1997).
[2] Pehkonen, E. (1997). The state-of-art in mathematical creativity. *ZDM Mathematics Education, 29* (3), 63-37.

## The 3Rs: Recite, Replicate, Regurgitate

Fractions Grade 6
Draw a rectangle with a 3 cm base and a 4 cm height. Shade 3/6th thereof.

Solution of a pupil:



While going through the classroom, that pupil asked me [the teacher] whether or not his solution was correct. *I was forced* to *admit that it was*. That is *what you get when you don't tell the pupils exactly what to do* . . . The teacher now reproaches himself for *not having prevented this solution.* He is obviously influenced by an insufficient understanding of what is mathematics, by the image of school as an institution for stuffing of brains . . . (p. 88)

Köhler, H. (1997). Acting artist-like in the classroom. *ZDM Mathematical Education, 29*(3), 88–93.

## 2007 bi-annual conference: International Community of Teachers of Mathematical Modeling and Applications
Co-hosts: Indiana University, Purdue University's INSPIRE, USAFA

"The Air Force Academy is a good example...where they came to us with a problem and they said **'our cadets come in here; they're smart kids. They come out knowing more, and they get worse on absolutely every scale of being good problem solvers, of being creative.'** They know more and can function less, in a way. And they're very worried, because **the person that they need** in the military just like other things, for the future, **isn't somebody who just follows rules. They need to understand those and be able to create their own flow of them.** So having them engaged helps us get on the forefront of things."

Richard Lesh, Professor of Learning Sciences, Cognitive Science, and Mathematics Education, Indiana University

## Teaching and Learning of Mathematics 1894*

The method of teaching should be throughout objective, and such as to call into exercise the pupil's mental activity. The text-books should be subordinate to the living teacher.

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## Teaching and Learning of Mathematics 2012

***Common Core State Standards for Mathematical Practice (NGA & CCSSO, 2010)***

1. Make sense of problems and preserve in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critiques the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

***Using the Common Core State Standards for Mathematics with Gifted and Advanced Learners (Johnsen & Sheffield, 2012)***

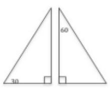
9. Solve problems in novel ways and pose new mathematical questions of interest to investigate.

## Key Shifts in the CCSS-M

- Greater focus on fewer topics
- Rigor: Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity
- Coherence: Linking topics and thinking across grades
  - Progressions Documents for the Common Core Math Standards
    - Work in progress at the University of Arizona's Institute for Mathematics and Education
      - explain why standards are sequenced the way they are,
      - point out cognitive difficulties and pedagogical solutions, and
      - give more detail on particularly knotty areas of the mathematics

## Impact of CCSS-M on Standardized Tests

1. Which of the following **best** describes the triangles shown below?




**A** both similar and congruent

**B** similar but not congruent

**C** congruent but not similar

**D** neither similar nor congruent

California Standards Test, released test questions, geometry, 2009

2. Triangle ABC undergoes a series of some of the following transformations to become triangle DEF:

- Rotation
- Reflection
- Translation
- Dilation

Is DEF always, sometimes, or never **congruent** to ABC? Provide justification to support your conclusion.

Common Core Smarter Balanced Grade 8 Sample Item, 2013

## Teaching and Learning of Mathematics 2012

| 21st Century Skills Math Map | |
|---|---|
| Mathematical Association of America<br>National Council of Teachers of Mathematics<br>The Partnership for 21st Century Skills | |
| Learning and Innovation Skills | **Creativity and Innovation:** Students use a wide range of techniques to create new and worthwhile ideas, elaborate, refine, analyze and evaluate their own ideas in order to improve and maximize creative efforts, and demonstrate originality and inventiveness. |
| | **Critical Thinking and Problem Solving:** Students reason effectively, use systems thinking and understand how parts of a whole interact …make judgments, decisions and solve problems in both conventional and innovative ways. |
| | **Communication and Collaboration: Students** know how to articulate thoughts and idea effectively…listen effectively to decipher meaning…(communicate) for a wide range of purposes. |

## Mindsets [The power of believing that you can improve]

- Theories of Intelligence
  - An unchangeable entity – a fixed mindset
  - A malleable quality that can be developed – a growth mindset
- "Research has shown that, even when students on both ends of the continuum show equal intellectual ability, their theories of intelligence shape their responses to academic challenge" (p. 251)

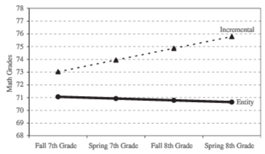



*Figure 1.* Graph of interaction effect of theory of intelligence and time on math achievement: Growth curves of predicted mathematics grades over 2 years of junior high school for students with incremental (+1 *SD* above the mean) and entity (−1 *SD* below the mean) theories of intelligence.

Blackwell, L.S., Trzesniewski, K. H. & Dweck, C. S. (2007) Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. Child Development, 78, 246-363. DOI: 10.1111/j.1467-8624.2007.00995.x

## Conversation

- Project Challenge (Chapin & O'Conner, Boston University)
- 4 year intervention focusing on discourse-based teaching in the lowest performing school district in Massachusetts
  - 400 4th graders, 70% low-income, 60% ELL
  - Teachers trained to use a variety of talk moves to encourage student to explain their reasoning and build on one another's thinking
  - After 2 years the proportion of student showing a "high probability of giftedness in mathematics" as measure on the Test of Mathematical Abilities (TOMA) rose from 4% to 41%
  - After 3 years 82% of Project Challenge students scores "Advanced" or "Proficient" on the state assessment (state average proficiency was 38%)

Resnick, L. B., Michaels, S. & O'Conner, M.C. (2010). How (well-structured) talk builds the mind R. J. Sternberg & Priess D.D. (Eds). *Innovations in Educational Psychology*, (163-194). New York: Springer.

## Creative Ability in Mathematics

The ability to

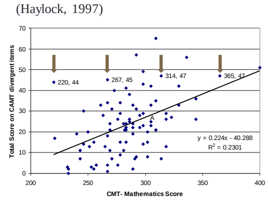
1. Formulate mathematical hypotheses
2. Determine patterns
3. Break from established mind sets to obtain solutions in a mathematical situation
4. Sense what is missing and ask questions
5. Consider and evaluate unusual mathematical ideas, to think through the consequences from a mathematical situation
6. Split general mathematical problems into specific sub-problems

Balka, D. S. (1974). Creative ability in mathematics. Arithmetic Teacher, 21, 633–636.



## Creativity in School Mathematics?

- An individual's knowledge base is the fundamental source of their creative thought. (Feldhusen and Westby, 2003)
- Students with equal mathematical achievement may have significant differences in performance on measures of mathematical creativity (Haylock, 1997)




Post Hoc Regression Analysis found that Achievement scores were a significant predictor of performance for those who scored below the mean creativity score but not above.

## Dan Meyer's: Math Class Needs A Makeover

Five symptoms that you're developing math reasoning skills wrong. Your students…

1. Lack initiative
2. Lack perseverance
3. Lack retention
4. Are adverse to word problems
5. Are eager for a formula

Two and Half Men approach to learning math:

- Teaching in small, "sitcom sized problems that wrap up is 22 minutes, 3 commercial breaks and a laugh track" resulting in **Impatient Problem Solvers**

## Dan's Recommendation

1. Use multimedia to bring the real world into the classroom
   - Dan often posts ideas and samples on his blog http://blog.mrmeyer.com/
2. Encourage student intuition
3. Ask the shortest question you can
4. Let students build the problem
5. Be less helpful

## Doing What Mathematicians Do

- **Mathematics** when it is finished, complete, all done, then it consists of proofs. But, when it is discovered, it **always starts with a guess**. . .
  **George Pólya (1966)**
- Mathematics - this may surprise you or shock you some - is never deductive in its creation. **The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions**. He arranges and rearranges his ideas, and he becomes convinced of their truth long before he can write down a logical proof. **The conviction is not likely to come early - it usually comes after many attempts, many failures, many discouragements, many false starts.**

  **Paul Halmos (1968)**

## Discovering the Art of Mathematics — Mathematical Inquiry in the Liberal Arts

- The **Discovering the Art of Mathematics**[1] project provides a wealth of resources to support college faculty in teaching Mathematics for Liberal Arts, including a library of 11 inquiry-based learning books, professional development opportunities, and extensive teacher resources.

**Current Titles:**

| | | |
|---|---|---|
| Art & Sculpture | Calculus | Dance |
| Games and Puzzles | Geometry | Knot Theory |
| Music | Number Theory | Patterns |
| The Infinite | Truth, Reasoning, Certainty and Proof | |

[1] This project is based upon work currently supported by the National Science Foundation under NSF1225915 and previously supported by NSF0836943 and a gift from Mr. Harry Lucas.

DISCOVERING THE ART OF MATHEMATICS
MATHEMATICAL INQUIRY IN THE LIBERAL ARTS

- Student Comments
  - Instead of falling asleep listening to lectures I was able to solve problems and make conjectures.
  - This course is a breath of fresh air. It helps me understand why math professors enjoy math so much. I see the fun in math now and how beautiful it can be.
  - This class taught me how to think independently about not only math but other subjects and everyday problem solving.
  - The fact that it was never easy to find an answer to the problem made me want to find it so much more.
  - Student Interviews on YouTube
    - https://www.youtube.com/watch?v=gKrRWt0zu0U
    - https://www.youtube.com/watch?t=11&v=LXb4JRbaIfk

## Negative Numbers

- Initially used only as a computational tool
- A controversial topic for centuries
  - Descartes (1637): False, fictitious numbers
  - Carnot (1803) to obtain an isolate negative quantity, it would be necessary to cut off an effective quantity from zero, to remove something from nothing: impossible operation.
  - Busset (1843)
    - attributed the "failure of the teaching of mathematics in France to the admission of negative quantities."
    - compelled to declare that ***such mental aberrations could prevent gifted minds from studying mathematics***

## Why is the product of two negative numbers positive?

- A fundamental question dealing with the properties of operations on numbers
  (- a x - b) = ab
  - Obvious that the product is ab but is it + or – ?
  - Cannot be - ab as (- a by + b) gives - ab and it can not have the same result
  - So, (- a by - b) must be positive

Euler,1770

## The Product of Negatives

- The Rule of Signs: To multiply a pair of numbers if both numbers have the same sign, their product is the product of their absolute values (their product is positive).
- Most often taught with
  - Direct instruction
  - Rules and Rhymes
  - Models (number lines, two-color chips)

## Does it make sense?

| Patterns | Inductive Reasoning |
|---|---|
| - 2 x 3 = - 6 | If - 1 x - 1 = - 1 then |
| - 2 x 2 = - 4 | - 1(1 + - 1) = - 1 x 1 + - 1 x - 1 |
| - 2 x 1 = - 2 | - 1(0) = - 1 + - 1 |
| - 2 x 0 = 0 | 0 = -2 |
| - 2 x - 1 = 2 | |
| - 2 x - 2 = 4 | **Algebraically** |
| - 2 x - 3 = 6 | (- 1 x - 1) + (- 1 x 1) = - 1(1 + - 1) |
| | (- 1 x - 1) + (- 1 x 1) = - 1(0) |
| | (- 1 x - 1) + - 1 + 1 = 0 + 1 |
| | - 1 x - 1 = 1 |

## THE STORY OF EMMA

A student at St Francis of Assisi, Notting Hill, UK

Classroom:
30 Students, 9 – 11 years old
Three different ability levels
Ethnically diverse: 25 out of 30 children are from non U.K. backgrounds
7 have English as a second language

2/7/13 27

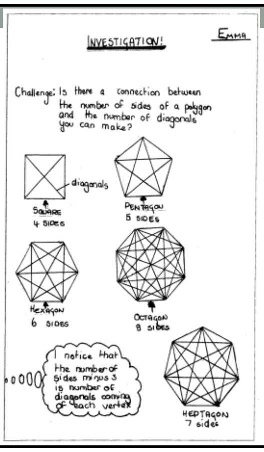
## What is your hypotheses?

Is there a connection between the number of sides of a polygon and the number of diagonals you can make?

Problem is loosely structured – a research question

Cause: change in number of sides
Effect: change in number of diagonals

## Emma's Work (Casey, 2011)



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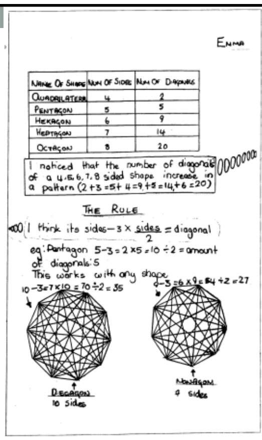


EMMA

| NAME OF SHAPE | NUM OF SIDES | NUM OF DIAGONALS |
|---|---|---|
| QUADRILATERAL | 4 | 2 |
| PENTAGON | 5 | 5 |
| HEXAGON | 6 | 9 |
| HEPTAGON | 7 | 14 |
| OCTAGON | 8 | 20 |

I noticed that the number of diagonals of a 4, 5, 6, 7, 8 sided shape increase in a pattern (2 + 3 = 5 + 4 = 9 + 5 = 14 + 6 = 20)

THE RULE

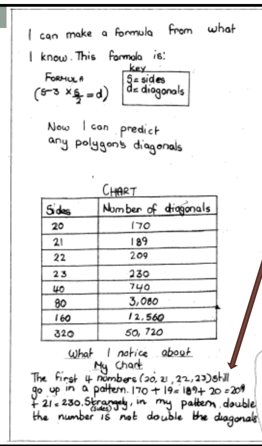
I think its sides − 3 × sides/2 = diagonal

eg: Pentagon 5 − 3 = 2 × 5 = 10 ÷ 2 = amount of diagonals: 5

This works with any shape



## Emma's Work



I can make a formula from what I know. This formula is:

FORMULA

(S − 3 × S/2 = d)

key
S = sides
d = diagonals

Now I can predict any polygons diagonals

CHART

| Sides | Number of diagonals |
|---|---|
| 20 | 170 |
| 21 | 189 |
| 22 | 209 |
| 23 | 230 |
| 40 | 740 |
| 80 | 3,080 |
| 160 | 12,560 |
| 320 | 50, 720 |

What I notice about My Chart

The first 4 numbers (20, 21, 22, 23) still go up in a pattern. 170 + 19 = 189 + 20 = 209 + 21 = 230. Strangely, in my pattern, double the number (sides) is not double the diagonals

There is a notation issue (S-3 × S/2 = d) will not generate the data chart Emma created.

The equation she used was (S-3) × S/2 = d

A teachable moment rather than a wrong answer.

"Strangely in my pattern double the number (sides) is not double the diagonals."

An opportunity to extend the problem with a student generated inquiry.

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## Emma's Voice

I tried to find the connection between the number of sides in each shape and the number of diagonals which were drawn. So I drew out a chart to help me look at the relationship between the numbers. I wondered what relation the numbers had to each other. When I looked at the number of diagonals in each shape I noticed that the difference between them increases by 1 each time. When I looked at the shape drawings again, I noticed that the numbers of diagonals coming from each vertex was three less than the number of sides in the shape eg. pentagon. 5 – 3 = 2 diagonals from each vertex, hexagon 6 – 3 = 3 etc. This gave me a rule for the number of diagonals coming from each vertex in any particular shape, but I needed to find the number of diagonals in each shape.

I tried to do what I thought of before. I tried to multiply the 2 × 5 (ie. the number of diagonals by the number of sides of the pentagon because there were five sides in the shape) and I noticed the answer was 10. I then noticed that 10 is twice the number of sides of a pentagon, and if I divided it by 2 it would give me the total number of diagonals, which it did. I then tried to do the same with the other shapes:

octagon (8 – 3 = 5) × 8 then divide the total by 2 = 20

nonagon (9 – 3 = 6) × 9 then divide the total by 2 = 27

This works with all the other shapes, it also allows me to predict the number of diagonals in any shape (Landers, 1999)

32

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Not search for a formula!

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Her first strategy

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Wondering and noticing not just punching numbers into a calculator in hopes that an answer will appear

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# The Voice of Emma’s Teacher

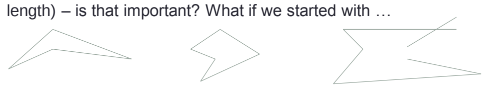
. . . from a starting point which involved all children in the class, provides evidence of what **children can achieve through being trained to think.** Previously, Emma would not have even attempted this investigation in spite of her mathematical talent. More importantly, I may not have given this kind of 'difficult' task to the class!

The class was **asked to investigate** if there was a connection between the number of sides of a polygon and the number of diagonals you can draw. I introduced this activity to the whole class**. Every child participated in the initial discussion** on the names and properties of polygons, in defining what diagonals are including demonstrations using people. Logo experiences were sought for further understanding of these concepts. From this common starting point, **children were encouraged to follow lines of enquiry** which matched their capability. What Emma has demonstrated here is a way of working she has acquired in the last few months.

**This way of working systematically and constantly refining her thoughts and processes has also enabled Emma to identify similar patterns in other problems and investigations**. For example, when we were investigating 'How many handshakes will take place if 20 people shook hands with each other?' Emma's comment was (within five minutes) 'This is similar to the polygons problem, I think. So I can cut down on quite a bit of working out'.

Is there a connection between the number of sides of a polygon and the number of diagonals you can make?

- Does starting with a triangle instead of a square make a difference?
- What if I asked some students about the relationship between the number of “corners” (vertices) rather than the number of “sides” (edges)?
- All the polygons shown were regular polygons – equiangular (all angles are equal in measure) and equilateral (all sides have the same length) – is that important? What if we started with …



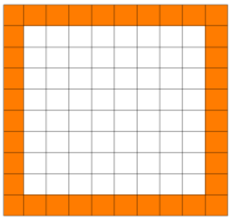
- Good problems (as opposed to exercises) often
  - generate more questions (lines of inquiry) and
  - Provide multiple entry points which
  - Offer more opportunities to differentiate

## The Border Problem

**Think** - Without counting every square, how can you determine how many squares are on the outside border of this 10-by-10 figure?

**Pair** – Share your strategy with your neighbor.

**Share** – We'll compare strategies as a group



## Student Developed Strategies*

| | |
|---|---|
| Jessica | 10 + 10 + 8 + 8 |
| Angelo | 4 x 10 – 4 |
| Daniel | 10 + 9 + 9 + 8 |
| Tibyan | 4 x 9 |
| Darian | 4 x 8 + 4 |
| Melissa | 10 x 10 - 8 x 8 |

*https://themathletes.wordpress.com/2013/10/07/the-border-problem/

## Multiple Strategies – Same Solution

10 + 10 + 8 + 8 = 36

4 x 10 – 4 = 36

10 + 9 + 9 + 8 = 36

4 x 9 = 36

4 x 8 + 4 = 36

10 x 10 - 8 x 8 = 36

## Extend and Generalize

- Imagine a 6 x 6 grid. Use two strategies different from your own to find out how many different squares are on the border.
- Now try a 50 x 50 grid and an *n x n* grid.
- Algorithm* - a procedure or formula for solving a problem
  - We have multiple strategies. Going beyond showing they all yield the same answer can you find another way to show they are equivalent

* The word derives from the name of the mathematician, Mohammed ibn-Musa al-Khwarizmi. The word "algorithm" is derived from the Latinization of his name, and the word "algebra" is derived from the Latinization of "al-jabr", part of the title of his most famous book, "Al-Kitab al-mukhtasar fi hisab al-jabr wa'l-muqabala" ("The Compendious Book on Calculation by Completion and Balancing"). Al-Khwarizmi wanted to go from the specific problems considered by the Indians and Chinese to a more general way of analyzing problems, and in doing so he created an abstract mathematical language which is used across the world today. http://www.storyofmathematics.com/islamic_alkhwarizmi.html

## A Generalized Solution

Let *n* = the number of squares in one row of an *n* x *n* grid

| | |
|---|---|
| 10 + 10 + 8 + 8 = 36 | $n+n+n-2+n-2=\mathbf{4n-4}$ |
| 4 x 10 – 4 = 36 | $\mathbf{4n-4}$ |
| 10 + 9 + 9 + 8 = 36 | $n+n-1+n-1+n-2=\mathbf{4n-4}$ |
| 4 x 9 = 36 | $4(n-1)=\mathbf{4n-4}$ |
| 4 x 8 + 4 = 36 | $4(n-2)+4=4n-8+4$<br>$4n-8+4=\mathbf{4n-4}$ |
| 10 x 10 - 8 x 8 = 36 | $n\times n-(n-2)\times(n-2)=n{\uparrow}2-(n{\uparrow}2-4n+4)$<br>$n{\uparrow}2-(n{\uparrow}2-4n+4)=\mathbf{4n-4}$ |

## Solve problems in novel ways and pose new mathematical questions of interest to investigate*

"Mathematics is the art of explanations. If you deny students the opportunity to engage in this activity—to pose their own problems, to make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs—you deny them mathematics itself."

Paul Lockhart, *The Mathematician's Lament*

* Using the Common Core State Standards for Mathematics with Gifted and Advanced Learners (Johnsen & Sheffield, 2012)